



AMYGDALA

$$z \mapsto z^2 + c$$

A Newsletter of Fractals & $\mathcal{M}$ (the Mandelbrot Set)

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Issue #28

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In This Issue...

Robert Hill's *Oddball Functions* is a commentary on a series of slides he sent to Amygdala three years ago, four of which constitute the current slide set (#28). This represents an earlier stage in Bob's work than *Dithering As A Shading Technique*, which appeared in Amygdala #27.

Ian Entwistle's *Julia Set Mappings with Decomposition* is an application to Julia sets, both conventional and unconventional, of a "decomposition" image-banding technique first discussed in *The Beauty of Fractals* (I believe). It is a special case ($n=2$) of the n -ary banding technique discussed by John Dewey Jones in *Decomposition: They Went Thataway — Another Way Of Viewing The Object* (Amygdala #13, pp 2-3), and nicely illustrated in slide A214 from slide set 13 and A209 from S15.

Carl Guthrie's *Complex Dimensions* arose from an idea of Carl's that, just as the two-dimensional set of complex numbers are an extension of the one-dimensional real numbers resulting from the inclusion of $\sqrt{-1}$, that the complex numbers themselves could be extended to higher dimensional number spaces by adjoining $\sqrt{i}$, $\sqrt{\sqrt{i}}$, and so on. Alas! this idea founders on the hard fact that the complex numbers are algebraically closed: $\sqrt{i}$, $\sqrt{\sqrt{i}}$, etc. are themselves complex numbers, and do not generate higher-dimensional analogs.

As often happens, however, Guthrie's idea produced interesting results even though its mathematical underpinnings are flawed.

Oddball Functions

—From Bob Hill's letter, dated October 5, 1989

I am enclosing a few slides (#2401-2412) which I hope you will find of interest. They are nearly all generated with functions other than $z^2 + c$. Virtually all the literature on fractals and related matters, including Amygdala, gives one the impression that there is only one Mandelbrot set, or at least only one of any particular interest. Even Devaney uses somewhat different language when discussing $\mathcal{M}$ -sets based on trigonometric and other functions and, for me at least, he somewhat obscures their relation to the original set of Mandelbrot. For example, his "explosion" of the sine Julia set I find identical to the changes encountered in passing from the connected $\mathcal{M}$ -set to the region immediately external to it.

I have found, however, that $\mathcal{M}$ -sets derived from a variety of functions are extremely interesting. In attempting to reproduce some of Devaney's pictures (*Science* 235, 342 (1987)), I expressed the trig functions as power series. Slides 2401-2403 are the $\mathcal{M}$ -sets so obtained for $\sin z$, using, respectively, 1, 2 and 3 terms of the series in z . Of course, in converting these rather innocent-looking expressions in z to the corresponding $x + iy$ expressions, they really lost their innocence! It is clear from the pic-

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tures that as more terms are added, more 'bulbs' will appear in the $\mathcal{M}$ -set, so that an infinitely long figure will be approached.

Slide 2404 is a similar picture of $\cos z$, using the 3-term series. Its general structure is not unexpected, after seeing $\sin z$.

Most of my early exploration of $\sin z$ was directed to the first "valley", corresponding to some of Devaney's pictures. My results were similar to his. However, by abandoning the emphasis on the sine function and considering these truncated series as polynomials in their own right, I realized that I should explore entire Julia sets, rather than just those portions near the origin, as done by Devaney. Slide 2407 is such a set, based on the 2-term sine series. I think you will agree that it introduces some aesthetic features not found in sets from $z^2 + c$.

Slides 2405 and 2406 are details of Julia sets based on the 3-term sine series. However, they are located on the perimeter of the $\mathcal{M}$ -set, quite distant from the region of Devaney's pictures. Again, these are quite unlike anything I have seen from $z^2 + c$.

All of the above examples were derived from expressions in z . One may also write expressions in x and y that are not easily expressed in z^1 . These can give strange results. The most satisfactory seem to be those functions possessing some symmetry among their terms. Thus, removing the coefficients from the 2-term sine series (2402) gives very startling results (2408), leading to Julia sets that are really quite different (2409).

Slides 2405 and 2406 exhibit another feature: the use of functions other than the conventional distance formula $x^2 + y^2$ to limit the escape time — what I call the *escape limit function*. Use of such functions often adds interest to otherwise featureless parts of a picture. Slide 2410 is an example of such a function, $x^2 - y^2 - 2xy$, applied to an area of the conventional $\mathcal{M}$ -set ($z^2 + c$).

1. And, in particular, that are not the real and imaginary parts of analytic functions, i.e. do not satisfy the Cauchy-Riemann conditions. RS

The last two slides are taken from other polynomial m -sets. Slide 2412 is a detail of the $\mathcal{M}$ -set for the function $z^5 + z^2 + c$, while 2411 is a Julia set from $z^4 + c$. The symmetry of Julia sets from simple powers of z is interesting in that the n^{th} power gives Julias with n -fold symmetry. This is in contrast to the corresponding $\mathcal{M}$ -sets, which show $(n-1)$ -fold symmetry².

The bad news arising from these experiments is that computation times mount up rapidly. On my Amiga 1000, computation of a high resolution picture of the 3-term sine function takes 30-40 hours. That is why some of these slides are low resolution. I need to get some good advice on how best to upgrade my computer so that these times might be shortened. Even so, I would be happy to provide hi-res versions of any of these pictures if you are interested.

All of these pictures were obtained using the program "Doug's Math Aquarium" from Seven Seas Software. I have found this program to be superb for visualization of all kinds of mathematical relationships. I can't recommend it too highly for Amiga owners.

You may have noticed that the slides are not standard 35mm slides. The standard 35mm format has an aspect ratio different from that of TV and computer monitor screens, so that to photograph the entire height of a screen, extraneous and possibly distracting areas on the sides must be included. Alternatively, just the width may be included, but areas at the top and bottom are cut off. Amygdala readers may be interested to know that slide mounts with the correct CRT aspect ratio are available from Heindl Masks'N'Mounts, Inc. / P.O. Box 150 / Hancock, VT 05748. They have many styles of mounts. #441 is the one to order; they are \$2 for a package of 15 or \$10 for a package of 100³.

I would also recommend the use of Polaroid "Presentation Chrome" 35mm film for photographing computer screens. This is a high contrast film

2. Papathomas and Julesz, *Visual Computer* 3, 23-26 (1987).

3. Or were, in 1989!



intended for making slides for presentations. It requires E-6 processing, the same as Ektachrome. Side by side comparison with Ektachrome shows that Presentation Chrome produces much superior color; no filtration is necessary or even desirable.

The Slides (S28)

All four of these slides are by Bob Hill. An extended comment on them can be found in his letter, found elsewhere in this issue under the title *Oddball Functions*.

A2405. A detail of a Julia set at $-.75+.75i$ based on

$$f(z) = c \left(z - \frac{z^3}{3!} + \frac{z^5}{5!} \right). \text{ Also uses a complex}$$

non-standard distance formula. Color incr depth = 1.56; DL=400; ER=16; Lo Res.

A2407. A detail of a Julia set at $0.1149+1.0658i$

$$\text{based on } f(z) = c \left(z - \frac{z^3}{3!} \right). \text{ Color incr}$$

depth = 4; DL=300; Escape limit function = $x^2 + y^2$; ER=16; Hi Res.

A2409. A detail of a Julia set at $1.1069+.39196i$ based on

$$f(z) = c \left(\left(x - \frac{x^3}{3!} + xy^2 \right) + \left(y + \frac{y^3}{3!} - x^2y \right) \right).$$

. Color incr depth = 1.5; DL=300; Escape limit function = $x^2 + y^2$; ER=16; Hi Res.

A2411. A detail of a Julia set at $-.3136+.467i$ based

on $f(z) = c + z^4$. Color incr depth = 0.625;

DL=200; Escape limit function = $x^2 + y^2$; ER=16; Hi Res.

Julia Set Mappings with Decomposition

Ian D Entwistle

Mappings of formulas such as $z_n = z_{n-1}^m + c$ are conventionally generated by assignment of a color to each point z_0 in a selected region of the complex plane on the basis of the number of iterations required for the absolute value of z_n to exceed a

preset limit Z . This means that the actual color of a point in the complex plane is not controlled by the specific value of Z but by the rate at which Z diverges. Other strategies for mapping the iterative behaviour of functions in the complex plane [Ent89, see References] do utilize the actual value of Z or of the real or imaginary parts of the modulus of the complex number to map the function. One of these which is not very widely utilised is *Binary Decomposition*. A number of quite striking illustrations of the method have appeared along with a highly mathematical description [Pei86]. As only a simple modification to the standard computer listing for obtaining Julia sets is needed to employ binary decomposition only a brief description is required to understand the method used to generate decomposition maps. For each iteration a complex number is obtained. At escape the modulus of this number (escape radius) or more usually its square is >4 . This complex number has a real and imaginary component and is frequently repre-

sented by the expression $Z = \sqrt{Z_{real}^2 + Z_{imag}^2}$. The complex number Z can also be represented as a vector and can be written in polar form as $Ze^{i\theta}$ where the angle θ is measured in a counterclockwise direction from the real axis, and is given by

$$\theta = \text{atan} \frac{Z_{imag}}{Z_{real}}. \text{ From the data that is generated}$$

during the iteration θ can thus be calculated. If the angle θ is less than 180° then the point is not plotted. If it is greater than 180° and less than 360° then it is plotted in color. This approach produces a typical binary decomposition plot as shown in Figure 1. Listing 1 shows the modification to a typical BASIC Julia set inner loop listing needed to produce the decomposition. It is important to note that the Level Set boundaries [Pei88] are visible and can still be used to control the color of the points. Simply include an extra statement to exclude certain iteration values from the decomposition test. This effect can be seen in Figure 1 where all points with iteration values greater than ten were unplotted. Figure 2 shows the effectiveness at higher magnification of decomposition: the Julia set outline is more easily seen by not plotting high value Level Set points [Jon90]. The squarish shapes of decomposition areas in two colour maps can be modified to change the appearance readily. In Fig-



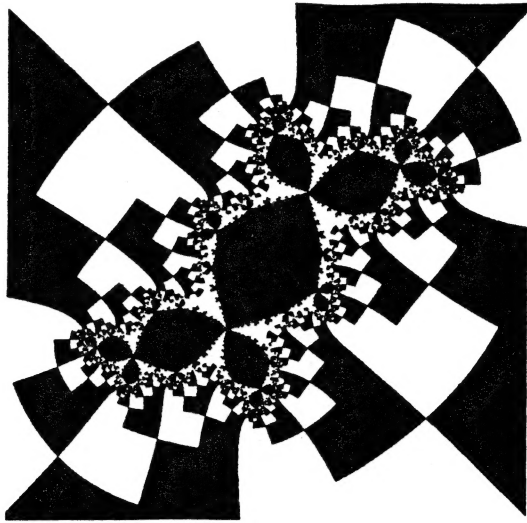


Figure 1: Typical Binary Decomposition

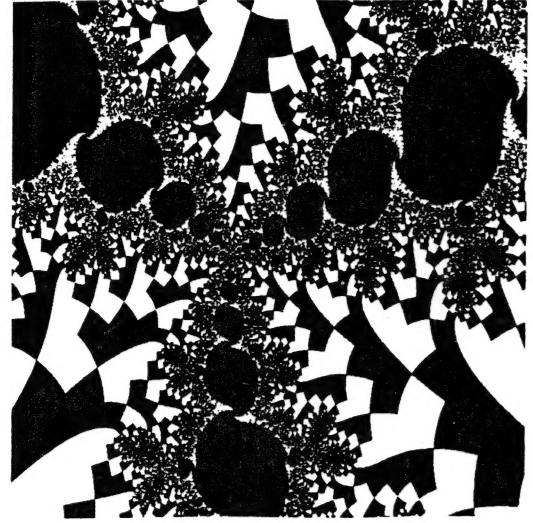


Figure 2: Effectiveness at higher magnification

ure 3 only the imaginary part of z^2 was used for the escape test. The program for this figure can be obtained by changing line 80 in Listing 1 to

```
IF Y1Y1>4 GOTO 110
```

An increase in control of the colour of the points in decomposition maps can also be effected by using *Multiple Decomposition* [Jon90, Ent92]. This requires an algorithm to check which of a larger number of angle ranges θ belongs to and then assigning a colour to these. The simplest method would just involve adding further tests to Listing 1, but an effective alternative is the variation given in

Listing 2. In line 150 the value of the angle θ is converted into an integer and this equated with a colour value as in the conventional mappings. One simple way to effect this for a black and white illustration is to assign 0 or 1 values to an array as alternatives and then equate the integer θ values with the array variables. This results in a mapping of the type shown in Figure 4. Note that the bounded points are plotted based on the minimum value of Z achieved during iteration using the variable $C1$. This is readily effected by the extra lines given in Listing 2, and uses a previously described method [Ent89]. The variable $MM\%$ in line 150 can be

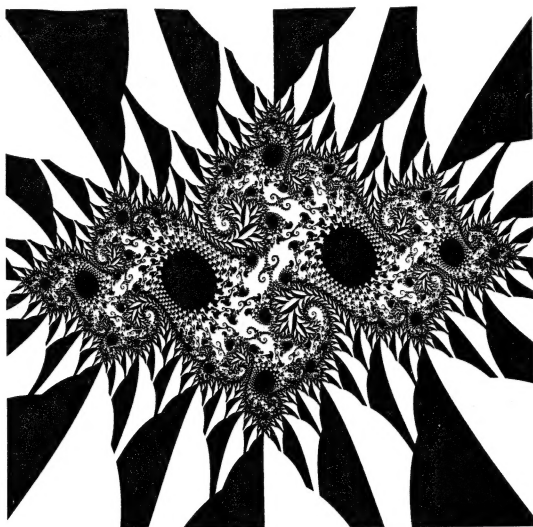


Figure 3: Escape Test Using Imaginary Part

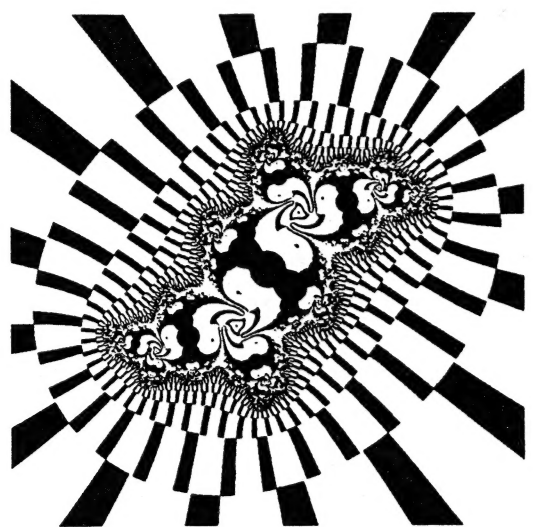


Figure 4: Using Minimum value of Z



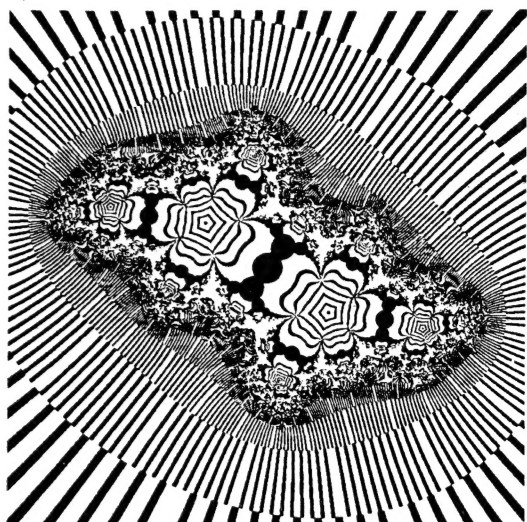


Figure 5: Increase In The Number Of Ranges

used to increase the number of ranges of the size of the angle θ . An increase from two up to ten results in the variation shown in Figure 5. In Figure 6 an additional variation is included. This utilises the escape test $z_{imag}^2 > 9$. In all these examples of decomposition the outlines of the level sets are visible. Some variation in colour control can be effected by alteration of the value of MM% for different level sets independently. All the examples shown in Figures 1 to 6 result from iteration of the function $z^2 + c$. Equally pleasing results can of course be obtained from other polynomial and also

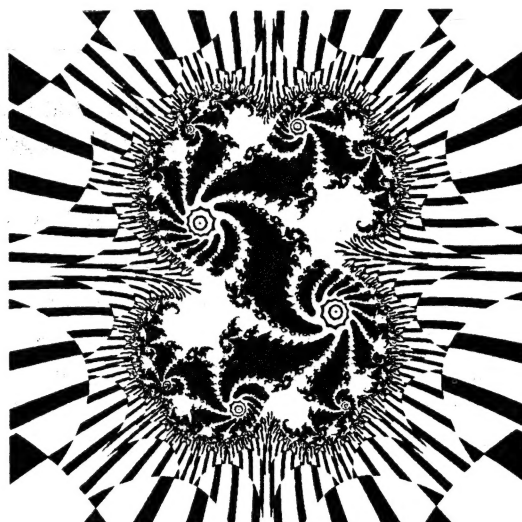


Figure 6: Using The Escape Test $z_{imag}^2 > 9$

geometric functions. For example Figure 7 was obtained by iteration of $z^5 + c$ using a further variation of decomposition mapping. Here the minimum values of the modulus of Z were again used to plot the points in the bounded set and the divergent points were obtained using Listing 3. In this program an experimental change was made to the calculation of θ . Instead of ARCTAN in line 150, $1/\text{TAN}$ was used. Listing 3 is obtained then by replacing line 150 in Listing 2 as indicated. Some adjustment was then necessary in the value of MM% in order to obtain a suitable range of integers for

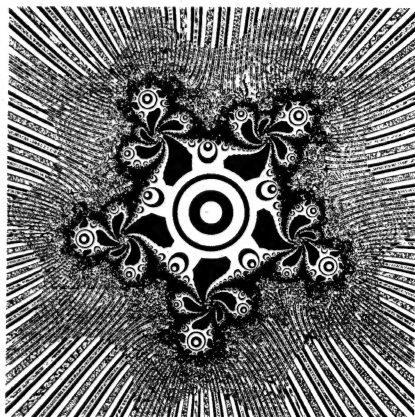


Figure 7: Iteration Using $z^5 + c$

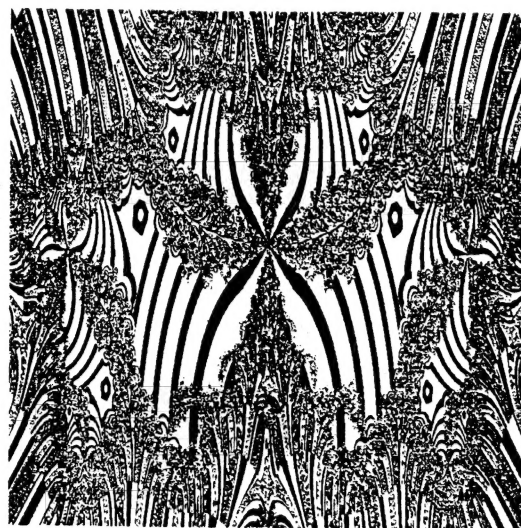


Figure 8: Iteration Using $z^4 - z + c$



assignment of colour. In addition note that the square escape test $|Y_1Y_1| > 4$ was used. The algebra for the iteration of $z^5 + c$ has been described in Amygdala (Issue 22, p5); also see [Ent89]. A fractal graphic derived from the same function $z^4 - z + c$ as that for Figure 8 appeared on the cover of Fractal Report 13. Now disguised by decomposition it is obtained by using the methods of Figure 7. The algebra for the inner loop is outlined in Scheme 1. For Figure 9 the algebra of $\sinh z$ (hyperbolic sine)

in Scheme 2 replaces the standard algebra for $z^2 + c$ Julia sets in Listing 3 and the tests used for Figure 7 were applied. The snakelike central fractal is the bounded set with all the points now plotted in black. A minimised version of this can be seen in [Ent89]. All the data needed to obtain Figures 1 through 8 are listed in the Table. The methods described here for the Julia set mappings can be applied as well to Mandelbrot set mappings for all the described functions.

Listing 1

```

20  FOR TI%=1 TO IT%           :REM ITERATIONS 1 TO MAX OF INNER LOOP
30  X1X1=X1*X1:Y1Y1=Y1*Y1:
40  XYZ=X1*Y1
50  X1=X1X1-Y1Y1+REALC
60  Y1=XYZ+XYZ+IMAGINARYC
70  CS= X1X1+Y1Y1
80  IF CS>4 GOTO 110
90  NEXT
100 GOTO 190                   :IE. EXIT LOOP AND PRINT POINT BLACK
110 IF TI%>10 THEN GOTO 190    :PLOT POINT WHITE
120 DELTA=Y1/X1                :DIVISION WITH SMALL FP ,WATCH FOR DIV BY 0 ERROR
130 THETA=ATN(ABS(DELTA))
140 IF X1<0 AND Y1>=0 THEN THETA +=1.5707963 :BINARY SELECTION
150 IF X1<0 AND Y1<0 THEN THETA +=3.14159625
160 IF X1>0 AND Y1<0 THEN THETA +=4.17123899
170 IF THETA >=0 AND THETA <=3.14159625 THEN GOTO 190
175 ASSIGN COLOUR : PRINT POINT WHITE
180 ASSIGN COLOUR AS WHITE : PRINT PIXEL
190 NEXT MI%
```

Listing 2

```

10  CI=25
20  FOR TI%=1 TO IT%
30  X1X1=X1*X1:Y1Y1=Y1*Y1:
40  XYZ=X1*Y1
50  X1=X1X1-Y1Y1+RC
60  Y1=XYZ+XYZ+IC
70  CS= X1X1+Y1Y1
80  IF CS>4 GOTO 130
90  IF CS<CI THEN CI=CS
100 NEXT
110 CI=INT(CI*FI%)             :REM FI%=INTEGER FACTOR
120 ASSIGN COLOUR TO CI VALUE : PRINT PIXEL : GOTO 160
130 IF TI%>10 THEN PRINT WHITE :GOTO 160
140 YX=Y1/X1
```



Figure 8: Julia Set Using $\sinh z$



```

150 THETA=INT (ABS (ATN (YX) *MM%)) :ASSIGN COLOUR
155 PRINT PIXEL :REM MM%=DECOMP. FACTOR
160 NEXT MI% :REM REAL PIXELS

```

Listing 3

```

150 THETA=INT (ABS (1/TAN (YX) ))MOD MM% :REM ALTERNATIVE FOR FIGURE 7

```

Scheme 1

```

340 XX=X*X:YY=Y*Y
350 X4=XX*(XX-6*YY)+YY*YY-X+RC
360 Y4=4*X*Y*(XX-YY)-Y+IC
370 X3=XX:Y3=YY

```

Scheme 2

```

340 XX=X*X:YY=Y*Y
341 AW=EXP (X) :WA=EXP (-X)
350 X4=.5*(WA-AW)*COS (Y) +RC
360 Y4=IC-.5*(WA+AW)*SIN (Y)

```

Fg	Xctr	YCtr	Mag	Creal	Cimag	Iter	Pixels	MM%	FI%
1	0.000000000000	0.000000000000	0.556	-0.12	+0.74	100	800x800	-	-
2	0.254898071300	0.158016204825	30.797	-0.107471875	+0.84765625	250	800x800	-	-
3	0.000000000000	0.000000000000	0.625	-0.74543	+0.11301	250	2400x2400	-	-
4	0.000000000000	0.000000000000	0.500	-0.11	+0.6667	100	800x800	2	50
5	0.000000000000	0.000000000000	0.588	-0.481762	-0.531657	100	800x800	10	100
6	0.000000000000	0.000000000000	0.625	+0.32	+0.043	100	800x800	2	50
7	0.000000000000	0.000000000000	0.667	+0.4	-0.68	100	1024x1024	40	150
8	-0.260000000000	0.000000000000	1.786	-0.78	+0.0	100	800x800	40	100
9	0.000000000000	-0.250000000000	0.606	-1.1666	-0.8086	120	1024x1024	40	100

Table 1: Window Parameters

References

- [Ent89] I. D. Entwistle, *Julia Set Art and Fractals in the Complex Plane*, Comput. & Graphics, 13(3) , pp 389-392,1989.
- [Pei86] H-O. Peitgen, P.H. Richter, *The Beauty of Fractals*, Springer-Verlag, 1986, Berlin.
- [Pei88] H-O. Peitgen , D.Saupe (Eds.), *The Science of Fractal Images*, Springer-Verlag,Berlin, 1988.
- [Jon90] J.D. Jones, *Three unconventional representations of the Mandelbrot set*, Comput. & Graphics, 14(1), pp 127-129,1990.
- [Ent92] I. D. Entwistle, *Contribution to The Pattern Book*, M & T Books Redwood City, C.A. 1992?

Complex Dimensions

Carl Guthrie

When I was introduced to the complex beauty of the Mandelbrot Set, my first question was: "What does it look like in three dimensions?" There are three-dimensional views available, but there is always a nagging doubt: Is this view pictured in the "true" third dimension? For that matter, is there such a thing as a true third dimension in complex numbers? Does the Set have real substance, or is it a two-dimensional shadow?

I have a degree in physics with a math minor and taught high school courses from basic math through calculus for six years, but this does not make me a mathematician. In my search for a third number axis I tried many approaches which



proved to be dead-ends. To avoid suspicion of complete ignorance I will not discuss them here. With help from Rollo Silver I think we have sorted out the mathematics for some interesting investigations, if not for a true multi-dimensional number system.

There are multidimensional number systems, but the unique properties of the Mandelbrot set are the result of the properties of complex numbers. Perhaps the mathematicians have multi-dimensional complex number systems preserved in their arcane journals, but, if so, they are not generally known. Numbers can be arbitrarily defined as higher dimensions, with arbitrarily defined connections to $a + bi$, and will yield figures for fractal generators, but these numbers lack the properties of complex numbers, and the figures generated with them will resemble the Mandelbrot Set only near the r and i axes. The fractal images I generated in such systems warp, truncate, and decompose badly as they get farther from the familiar axes, or they merely rotate the fractal image about the real axis. If a third or higher dimension produces a figure as truly complex as the Mandelbrot set, it must have similar properties to the familiar complex number system. This produces a mathematical quandary: the complex numbers are algebraically closed, so that any operation on complex numbers will produce complex numbers. All roots of -1 can be reduced to the form $a + bi$, so that all axes of a higher dimensional number system formed from roots of -1 are only lines through the origin in the same old complex plane. This seems at first to preclude using other roots of -1 as "dimensions", but actually this gives us complex numbers to use as our number axes, and is what we want. As the complex plane itself is created by definition, we can define complex space.

Although I have a degree in physics with a math minor, and have taught high school courses from basic math through calculus, I am not a mathematician. In my search for a third number axis, I tried many mathematically embarrassing dead-ends. To avoid the charge of contempt of mathematics, we will not speak of those things here. With help from Rollo Silver, I think we have sorted out the mathematics for some interesting investigations, if not for a true multidimensional system.

As the real and imaginary numbers form a two-dimensional system, let the n th root of -1 be the basis of an n -dimensional number system. To illustrate, take the normal complex numbers:

Let $i = \sqrt{-1}$.

Then

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

	Axis	A	B
		1	i
A		1	i
B		i	-1

Product Table

Figure 9: Complex Arithmetic

We do not have to define mathematically a new number to use as the basis of a multidimensional number system. If we define the n th root of -1 as the basis of an n -dimensional number system, all numbers generated in this "space" can be reduced to the form $a + bi$, but there is no saving in computer operations by doing so. I find it less confusing to call the n th root of -1 a separate dimension. The development of a four-dimensional number system follows the same pattern as the two-dimensional system above:

Let $j = \sqrt[4]{-1}$

Then

$j^0 = 1$

$j^1 = j$

$j^2 = i$

$j^3 = k$

$j^4 = -1$

$j^5 = -j$

$j^6 = -i$

$j^7 = -k$

$j^8 = 1$

	Axis	A	B	C	D	
		1	j	i	k	-1
A		1	j	i	k	-1
B		j	i	k	-1	$-j$
C		i	k	-1	$-j$	$-i$
D		k	-1	$-j$	$-i$	$-k$
		-1	$-j$	$-i$	$-k$	+1

Product Table

Figure 10: 4D Arithmetic

It is assumed that the reader has a knowledge of the mechanics of generating the Mandelbrot set. Development of the generator of the Mandelbrot set in four dimensions is as follows: $z_{n+1} \rightarrow z_n^2 + z_0$



(where $z = A + Bj + Ci + Dk$, $z_1 = 0$, and z_0 is the point being examined. Expansion of the generator yields the following components of z_{n+1} :

$$A_{n+1} = A_n^2 - 2B_nD_n - C_n^2 + A_0$$

$$B_{n+1} = 2A_nB_n - 2C_nD_n + B_0$$

$$C_{n+1} = 2A_nC_n + B_n^2 - D_n^2 + C_0$$

$$D_{n+1} = 2A_nD_n + 2B_nC_n + D_0$$

$$|z|^2 = A^2 + B^2 + C^2 + D^2$$

z_0 is assumed to be a divergent point if $|z_{n+1}|^2 > 4$ for some value of n , or convergent if it converges or does not diverge within a given number of iterations of the generator. The number of iterations to produce convergence or divergence is the "value" of the point. All components must be calculated for each iteration, even for sections of only two dimensions.

There are six cross-sectional planes in the 4D system. In the cross-sections through the origin we find the following:

- Section AB is identical to Section AD. Figure 11 gives an example, with $-2 \leq A \leq 2$, $-1.5 \leq B \leq 1.5$, $C_0 = D_0 = 0$.
- Section AC is the familiar Mandelbrot Set.
- Sections BC and DC are identical, but inverted around C.
- Section BD gives the most interesting views, two of which are given in Figures 4 and 5. Figure 12 has $-1 \leq B \leq 1$, $-0.75 \leq D \leq 0.75$,

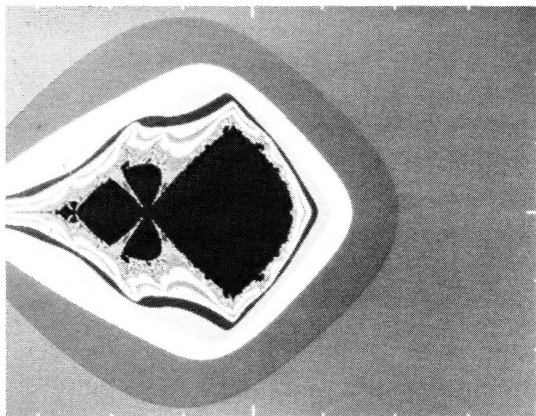


Figure 11: The Flaming Bishop

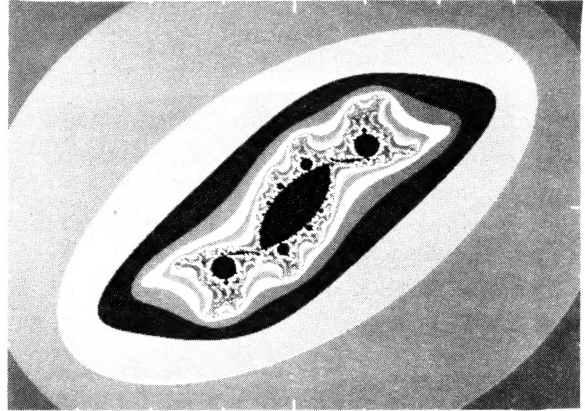


Figure 12: Section BD

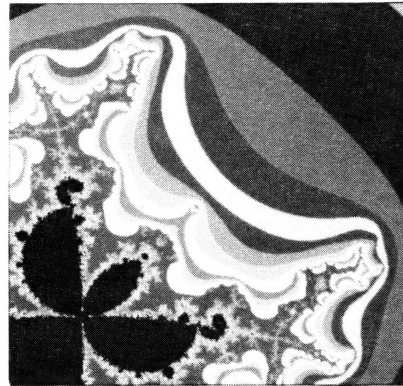


Figure 13: Smiling Clam

$$A_0 = 0.3, B_0 = 0.2.$$

Figure 13 has: $0.3 \leq B \leq 0.6$, $0.3 \leq D \leq 0.6$, $A_0 = 0.25$, $B_0 = 0$. It is a 10x view of another picture for which $-1.5 \leq B \leq 1.5$, $-1.5 \leq D \leq 1.5$, $A_0 = 0.25$, $B_0 = 0$.

Any number of dimensions can be defined, and calculations developed, in the same manner as for four dimensions. Cross-sections can be investigated at any point on any axis. I have investigated systems of four, eight, sixteen, three, five, and six dimensions. Twelve dimensions is the next project, and it shows great promise. The number of possibilities is mind-boggling.

Odd numbers of dimensions are disappointing, so far. Five dimensions was a disaster — if I programmed it correctly. The three-dimensional system created by taking the cube root of -1 does not produce an esthetically satisfying "Mandelbrot



object", but raises some intriguing mathematical questions. The 3D system which uses the real number line as one axis does not contain the i -axis, and results in a lopsided and truncated Mandelbrot set in each of its three cross-sectional planes. Although the three axes, when reduced to the three cube roots of -1 in the complex plane, lie 120° apart, the warping and truncating in the three axial cross-sectional planes are on an angle of 45° to the axes.

Why should there be the truncated, planar division between convergent and divergent points on the southeast-front side of the Mandelbrot object? This area is the only place investigated where the boundary between convergent and divergent points is apparently simply a plane. A 3D system which contains the i -axis will not contain the real number line. This system is yet to be investigated — if it is mathematically possible to define.

The convergent points in all multidimensional systems exhibit order, symmetry, and a "social structure" of values, that are fascinating. Rings of alternating values in one cross-section produce interesting artifacts, at low magnification. Assignment of colors to these views can produce esthetically very pleasing pictures.

The original purpose of these investigations was to develop a perspective view of the "Mandelbrot object in three dimensions. I have delayed this chore for four reasons: (1) lack of high-powered computing hardware for the extensive calculations; (2) lack of satisfactory software for speed and color support; (3) lack of programming expertise to improve conditions in (1) and (2); and (4) the cross-sections are such a rich field that it is easy to stay with them, making random views of sights never before seen by mortal eyes. Perhaps a reader of this article will develop a satisfactory 3D perspective view before I do. The four-dimensional system will probably produce quicker and more satisfying results. Good luck!

Fading Away?

Well, we're down to 508 subscribers — 20 less than last issue. If we continue in this downward fashion, we'll pass a fatal point where I'll no longer be able to afford to continue editing and publishing Amygdala! I need your help, subscribers! As I

asked in the previous issue: Are people losing interest in fractals? Is it because of the long hiatus in publication over the past year? Has this newsletter become irrelevant or boring? Those questions could only be answered by taking a market survey, if at all, so I'll regard them as unanswerable, except by each of me/you for him/herself.

I'll assume that many of you are interested in keeping Amygdala going. For that to happen I'm going to need your help to boost circulation. I guess I'm a pretty good editor, but I'm a lousy marketing and sales person. I believe there are several thousand potential subscribers out there; the problem is, how to reach them. I'm not interested in "selling" them, just in getting them to know about Amygdala. Here's what you can do:

1. Get fractal friends to subscribe.
2. If you publish a book or periodical, think about mentioning Amygdala in it.
3. If you are active on electronic forums or bulletin boards where fractals are of interest, please mention Amygdala there.
4. If you read publications in which fractals are a topic, send a letter mentioning Amygdala.
5. If you're sharing your issues with someone else, get them to subscribe on their own. Saving money by piggybacking may be appropriate for big, rich publications like the Wall Street Journal, but it may just serve to kill the Amygdalan goose.

Circulation

As of 23 August 1992 Amygdala has 508 subscribers, 165 of whom have the supplemental color slide subscription.

Renewal

For 95 of you subscribers out there this is the last issue of your current subscription. You can determine if you are one of those by looking at the lower left of your mailing label: If it reads "A28" this is your last issue. if it reads "S28" this is your last slide set.

I urge you to use the enclosed form to renew your subscription promptly to avoid missing anything.

